To the Right Honourable and others, the Learned Members of the Royal Society, for the Advancement of the Sciences.

PResenteth to your Consideration, your most humble servant *Thomas Hobbes*, a Confutation of a Theoreme which hath a long time passed for Truth; to the great hinderance of Geometry, and also of Natural Philosophy, which thereon dependeth.

The Theoreme.

The four sides of a Square being divided into any number of equal parts, for example into 10; *and straight lines drawn through the opposite points, which will divide the Square into* 100 *lesser Squares; The received Opinion, and which Dr. Wallis commonly useth, is, that the root of those* 100, *namely* 10, *is the side of the whole Square.*

The Confutation.

The Root 10 *is a number of those Squares, whereof the whole containeth* 100, *whereof one Square is an Unitie; therefore the Root* 10, *is* 10 *Squares: Therefore the Root of* 100 *Squares is* 10 *Squares, and not the side of any Square; because the side of a Square is not a Superficies, but a Line. For as the root of* 100 *Unities is* 10 *Unities, or of* 100 *Souldiers* 10 *Souldiers: so the root of* 100 *Squares is* 10 *of those Squares. Therefore the Theoreme is false; and more false, when the root is augmented by multiplying it by other greater numbers.*

Hence it followeth, that no Proposition can either be demonstrated or confuted from this false Theoreme. Upon which, and upon the Numeration of Infinites, is grounded all the Geometry which Dr. *Wallis* hath hitherto published.

And your said servant humbly prayeth to have your Judgement hereupon: And that if you finde it to be false, you would be pleased to correct the same; and not to suffer so necessary a Science as Geometry to be stifled, to save the Credit of a Professor.

Three PAPERS

Prefented to the

ROYAL SOCIETY
Againſt Dr. *WALLIS*.

Together with

CONSIDERATIONS
ON
Dr. *Wallis* his ANSWER to them.

By THO. HOBBES of *Malmsbury*.

LONDON:
Printed for the Author; and are to be had at the Green
Dragon without *Temple-bar*. 1671.

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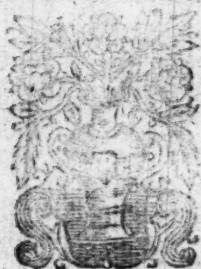
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By THOMAS HOBBES of Malmsbury.

To the Right Honourable and others, the Learned Members of the Royal Society, for the Advancement of the Sciences.

Your most humble servant *Thomas Hobbes* presenteth, That the quantity of a Line calculated by extraction of Roots, is not to be truely found. And further presenteth to you the Invention of a Straight Line equal to the Arc of a Circle.

A Square Root is a number which multiplied into it self produceth a num-*Definition.* ber. And the number so produced is called a Square number. For example : Because 10 multiplied into 10 makes 100 ; the Root is 10, and the Square number 100.

Consequent.

In the natural row of Numbers, as 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, &c. every one is the Square of some number in the same row. But Square numbers (beginning at 1) intermit first two numbers, then four, then six, &c. So that none of the intermitted numbers is a Square number, nor has any Square root.

PROP. I.

A Square root (speaking of quantity) is not a Line, such as *Euclide* defines, without Latitude, but a Rectangle.

Suppose A B C D be the Square, and AB, BC, CD, DA be the sides ; and every side divided into 10 equal parts, and Lines drawn through the opposite points of division ; there will then be made 100 lesser Squares, which taken all together are equal to the Square ABCD. Therefore the whole Square is 100, whereof one Square is an Unit ; therefore 10 Units, which is the Root, is ten of the lesser Squares, and consequently has Latitude ; and therefore it cannot be the side of a Square, which according to *Euclide* is a Line without Latitude.

Consequent.

It follows hence, that whosoever taketh for a Principle, That a Side of a Square is a meer Line without Latitude, and That the Root of a Square is such a Line, (as Dr. *Wallis* continually does) demonstrates nothing.

But if a Line be divided into what number of equal parts soever, so the Line have bredth allowed it, (as all Lines must, if they be drawn) and the length be to the bredth as Number to an Unite, the Side and the Root will be all of one length.

PROP.

PROP. II.

Any Number given is produced by the greatest Root multiplied into it self, and into the remaining Fraction. Let the Number given be two hundred Squares, the greatest Root is $14\frac{4}{14}$ Squares. I say, that 200 is equal to the product of 14 into it self, together with 14 multiplied into $\frac{4}{14}$. For 14 multiplied into it self, makes 196. And 14 into $\frac{4}{14}$ makes $\frac{56}{14}$, which is equal to 4. And 4 added to 196 maketh 200; as was to be proved.

Or take any other Number 8, the greatest Root is 2; which multiplied into it self is 4, and the Remainder $\frac{4}{2}$ multiplied into 2, is 4. and both together 8.

PROP. III.
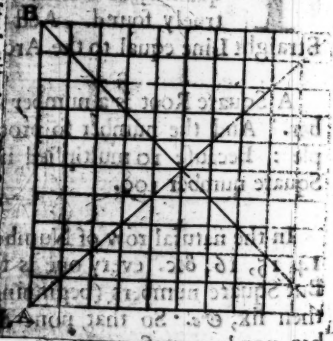
But the same Square calculated Geometrically by the like parts, consisteth (by *Eucl.* 2. 4.) of the same numeral great Square 196, and of the two Rectangles under the greatest side 14, and the Remainder of the side, or (which is all one) of one Rectangle under the greatest side, and double the Remainder of the side; and further of the Square of the less Segment; which all together make 200, and moreover $\frac{16}{49}$ of those 200 Squares; as by the operation it self appeareth thus.

The side of the greater Segment is, $14\frac{4}{14}$

$14\frac{4}{14}$

Which multiplied into it self, makes 200

The product of 14 the greatest segment, into the two Fractions, $\frac{4}{14}$, that is, into $\frac{8}{14}$ (or into twice $\frac{4}{14}$) is $\frac{56}{14}$ (that is 4) and that 4 added to 196 makes 200.

Lastly, the product of $\frac{4}{14}$ into $\frac{4}{14}$ or $\frac{2}{7}$ into $\frac{2}{7}$, is $\frac{16}{49}$. And so the same Square calculated by Roots, is less by $\frac{16}{49}$ of one of those two hundred Squares, then by the true and Geometrical Calculation; as was to be demonstrated.

Consequent.

It is hence manifest, That whosoever calculates the length of an Arc or other Line by the extraction of Roots, must necessarily make it shorter then the truth, unless the Square have a true Root.

The

The Radius of a Circle is a Mean Proportional between the Arc of a Quadrant and two fifths of the same.

DEscribe a Square ABCD, and in it a Quadrant DCA. In the side DC take DT two fifths of DC; and between DC and DT a Mean Proportional DR; and describe the Quadrantal Arcs RS, TV.

I say, the Arc RS is equal to the ſtreight line DC.

For ſeeing the proportion of DC to DT is duplicate of the proportion of DC to DR, it will be alſo duplicate of the proportion of the Arc CA to the Arc RS; and likewiſe duplicate of the proportion of the Arc RS to the Arc TV.

Suppoſe ſome other Arc leſs or greater then the Arc RS to be equal to DC, as for example *rs*. Then the proportion of the Arc *rs* to the ſtreight line DT will be duplicate of the proportion of RS to TV, or DR to DT. Which is abſurd; becauſe D*r* is by conſtruction greater or leſs then DR.

Therefore the Arc RS is equal to the ſide DC. Which was to be demonſtrated.

Corol.

Hence it follows that DR is equal to two fifths of the Arc CA. For RS, TV, DT being continually proportional; and the Arc TV being deſcribed by DT, the Arc RS will be deſcribed by a ſtreight line equal to TV. But RS is deſcribed by the ſtreight line DR. Therefore DR is equal to TV, that is, to two fifths of CA.
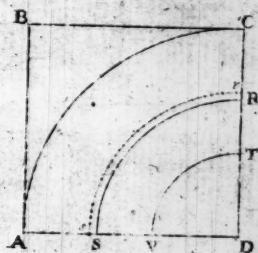
And your ſaid ſervant moſt humbly prayeth you to conſider (if the demonſtration be true and evident) whether the way of objecting againſt it by Square Roots, uſed by Dr. *Wallis*; and whether all his Geometry, as being built upon it, and upon his ſuppoſition of an Infinite Number, be not falſe.

Considerations upon the Answer of Dr. Wallis to the Three Papers of Mr. Hobbes.

DR. *Wallis* sayes, All that is affirmed, is but, *If we SUPPOSE That, This will follow.*

But it seemeth to me, that if the Supposition be impossible, then that which follows will either be false, or at least undemonstrated.

First, this Proposition being founded upon his *Arithmetica Infinitorum*, If there he affirm an absolute Infiniteness, he must here also be understood to affirm the same. But in his 39th Proposition he saith thus: *Seeing that the number of terms increasing, the excess above sub-quadruple is perpetually diminished; so as at last it becomes less than any proportion that can be assigned; If it proceed in infinitum it must utterly vanish. And therefore if there be propounded an Infinite row of quantities in triplicate proportion of quantities Arithmetically proportional (that is, according to the row of Cubical numbers) beginning from a point or 0; that row shall be to a row of as many, equal to the greatest, as 1 to 4.*

It is therefore manifest that he affirms, That in an Infinite row of quantities the last is given; and he knows well enough that this is but a shift.

Secondly, he sayes, That usually in *Euclide* and all after him, by *Infinite* is meant but, more than any assignable *Finite*, or the greatest possible.

I am content it be so interpreted. But then from thence he must demonstrate those his conclusions, which he hath not yet done. And when he shall have done it, not only the Conclusions, but also the Demonstration will be the same with mine in *Cap.* 14. *Art.* 2, 3, &c. of my Book *De Corpore*. And so he steals what he once condemn'd. A fine quality.

Thirdly, he sayes (by *Euclides* 10th Proposition, but he tells not of what Book) That a Line may be bisected, and the halves of it may again be bisected, and so onwards infinitely; and that upon such supposed Section Infinitely continued, the parts must be supposed Infinitely many.

I deny that; for *Euclide*, if he sayes a Line may be divisible into parts perpetually divisible, he means, That all the divisions, and all the parts arising from those divisions, are perpetually Finite in number.

Fourthly, he sayes, That there may be supposed a row of quantities infinitely many, and continually increasing, whereof the last is given.

'Tis true, a man may say (if that be supposing) that white is black; but if *Supposing* be *Thinking*, he cannot suppose an Infinite row of quantities whereof the last is given. And if he say it, he can demonstrate nothing from it.

Fifthly, He sayes (for one absurdity begets another) *That a Superficies or Solid may be supposed so constituted, as to be Infinitely long, but Finitely great*

(the

(*the breadth continually decreasing in greater proportion than the length increaseth*) *and so as to have no center of gravity.* Such is Toricellio's Solidum Hyperbolicum acutum, *and others innumerable discovered by Dr.* Wallis, *Monsieur* Fermat, *and others. But to determine this, requires more of Geometry and Logick (whatsoever it do of the Latine Tongue) than Mr.* Hobbes *is master of.*

I do not remember this of *Toricellio*, and I doubt Dr *Wallis* does him wrong, and Monsieur *Fermat* too. For to understand this for sense, 'tis not required that a man should be a Geometrician or a Logician, but that he should be mad.

In the next place he puts to me a Question as absurd as his Answers are to mine. Let him ask himself (saith he) if he be still of opinion, *That there is no Argument in Natural Philosophy to prove that the World had a beginning*: First, whether in case it had no beginning, there must not have passed an Infinite number of years before Mr. *Hobbes* was born. Secondly, whether at this time there have not passed more, that is, more than that Infinite number. Thirdly, whether, in that Infinite (or more than Infinite) number of years, there have not been a greater number of dayes and hours, and of which hitherto the last is given. Fourthly, whether, if this be an Absurdity, we have not then (contrary to what Mr. *Hobbes* would perswade us) an Argument in Nature to prove the World had a beginning.

To this I answer, not willingly, but in service to the Truth, that by the same Argument he might as well prove that God had a beginning. Thus: in case he had not, there must have passed an Infinite length of time before Mr. *Hobbes* was born; but there hath passed at this day more than that Infinite length (by eighty four years). And this day, which is the last, is given. If this be an Absurdity, have we not then an Argument in Nature to prove that God had a beginning? Thus 'tis when men intangle themselves in a Dispute of that which they cannot comprehend. But perhaps he looks for a Solution of his Argument to prove that there is somewhat greater than Infinite; which I shall do so far, as to shew it is not concluding. If from this day backwards to Eternity be more than Infinite, and from Mr. *Hobbes* his birth backwards to the same Eternity be Infinite, then take away from this day backwards to the time of *Adam*, which is more than from this day to Mr. *Hobbes* his birth, then that which remains backwards must be less than Infinite. All this arguing of Infinites is but the ambition of School-boyes.

To the latter part of the first Paper.

There is no doubt, if we give what Proportion we will of the Radius to the Arc, but that the Arc upon that Arc will have the same Proportion. But that is nothing to my Demonstration. He knows it, and wrongs the Royal Society in presuming they cannot find the Impertinence of it.

My.

My proof is this; That if the Arc on TV, and the Arc RS, and the ſtreight Line CD, be not equal, then the Arc on TV, the Arc on RS, and the Arc on CA, cannot be proportional. Which is manifeſt by ſuppoſing in DC a leſs than the ſaid DC, but equal to RS, and another ſtreight Line, leſs than RS, equal to the Arc on TV; and any body may examine it by himſelf.

I have been asked by ſome that think themſelves Logicians, Why I proceeded upon ⅖ rather than any other part of the Radius. The reaſon I had for it was, That long ago ſome *Arabians* had determined, That a ſtreight Line whoſe ſquare is equal to 10 ſquares of half the Radius, is equal to a quarter of the Perimiter; but their demonſtrations are loſt. From that Equality it follows, that the third proportional to the Quadrant and Radius, muſt be a mean proportional between the Radius and ⅖ of the ſame. But my anſwer to the Logicians was, That though I took any part of the Radius to proceed on, and lighted on the Truth by chance, the Truth it ſelf would appear by the Abſurdity ariſing from the denial of it. And this is it that *Ariſtotle* meant, where he diſtinguiſheth between a Direct demonſtration, and a demonſtration leading to an abſurdity. Hence it appears, that Dr. *Wallis* his objections to my *Roſetum* are invalid, as built upon Roots.

To the ſecond Paper.

First, he ſayes, That it concerns him no more than other men. Which is true. I meant it againſt the whole Herd of them who apply their *Algebra* to *Geometry*.

Secondly, He ſayes, That a bare Number cannot be the Side of a Square figure.

I would know what he means by a Bare Number. Ten Lines may be the ſide of a Square figure. Is there any Number ſo bare, as by it we are not to conceive or conſider any thing numbred? Or by ten Nothings underſtands he Bare 10? He ſtruggles in vain, his Conſcience puzzles him.

Thirdly, He ſayes, Ten Squares is the Root of 100 Square-ſquares. To which I anſwer, firſt, That there is no ſuch Figure as a Square-ſquare. Secondly, That it follows hence that a Root is a Superficies, for ſuch is 10 Squares.

Laſtly, He ſayes, That neither the Number 10, nor 10 Souldiers is the Root of 100 Souldiers; becauſe 100 Souldiers is not the Product of 10 Souldiers into 10 Souldiers.

That laſt I grant, becauſe nothing but Numbers can be multiplied into one another. A Souldier cannot be multiplied by a Souldier. But no more can a Square-figure by a Square-figure, though a Square-number may. Again, If a Captain will place his hundred Men in a ſquare Form, muſt not he take the Root of 100 to make a Rank or File? And are not thoſe 10 Men?

To

To the third Paper.

He objects nothing here, but that, *The Side of a Square is not a Superficies but a Line*, and that *a Square Root (speaking of quantity) is not a Line but a Rectangle*, is a contradiction. The *Reader* is to judge of that.

To his Scoffings I say no more, but that they may be retorted in the same words, and are therefore childish.

And now I submit the whole to the Royal Society, with confidence that they will never ingage themselves in the maintenance of these Unintelligible Doctrines of Dr. *Wallis*, that tend to the suppression of the Sciences which they endeavour to advance.

To the fourth Paper.

...hath, he hath; That it concerns him no more than to have read, which is to...

Evident it is, whole the whole Hold, of thing whatsoever, that Algebra to Ar...

Secondly, he says, That a Just Number cannot be the Side of a Square figure.

I would know what he means by a Just Number. If it be small Number, If it...

Thirdly, he says, Ten Square is the Root of 100 Square. To which I answer first, That there is no such Figure as a Square-figure. Secondly, That it follows hence that a Root is a Superficies, for 100 is 10 Square. Lastly, He says, That neither the Number 10, nor 10 Foot, may go ... is the Root of 100 Souldiers, because 100 Souldiers is neither Root nor File of Souldiers.

That this latter is true, because nothing but Numbers can be multiplied into one another. A Souldier cannot be multiplied by a Souldier. But no more can a Square-figure by a Square-figure, (though a Square-number may.) Again, If a Square will place his hundred Men in a square Form, must not he also the Root of 100 to make a Rank or File? And are not 100 to 10 Men?